TABLE II. The $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ (99.82\%).

| $P$ (bar) | $\beta\left(\mathrm{bar}^{-1}\right) \times 10^{6}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $V\left(\mathrm{~cm}^{3} \mathrm{~g}^{-1}\right)$ |  | $\alpha\left(\mathrm{deg}^{-1}\right) \times 10^{6}$ |
| $t=5{ }^{\circ} \mathrm{C}$ |  |  |  |
| 0 | 0.904468 | 51.547 | -114.14 |
| 100 | 0.899880 | 50.177 | -76.14 |
| 200 | 0.895436 | 48.846 | -40.21 |
| 300 | 0.891130 | 47.553 | -6.27 |
| 400 | 0.886959 | 46.297 | 25.75 |
| 500 | 0.882916 | 45.077 | 55.91 |
| 600 | 0.878997 | 43.891 | 84.29 |
| 700 | 0.875199 | 42.738 | 110.93 |
| 800 | 0.871515 | 41.618 | 135.91 |
| 900 | 0.867943 | 40.529 | 159.28 |
| 1000 | 0.864478 | 39.470 | 181.10 |
| $t=25^{\circ} \mathrm{C}$ |  |  |  |
| 0 | 0.905429 | 46.480 | 191.74 |
| 100 | 0.901282 | 45.326 | 206.31 |
| 200 | 0.897257 | 44.213 | 220.47 |
| 300 | 0.893347 | 43.137 | 234.19 |
| 400 | 0.889548 | 42.099 | 247.42 |
| 500 | 0.885856 | 41.095 | 260.13 |
| 600 | 0.882266 | 40.124 | 272.28 |
| 700 | 0.878774 | 39.185 | 283.85 |
| 800 | 0.875378 | 38.277 | 294.81 |
| 900 | 0.872072 | 37.397 | 305.14 |
| 1000 | 0.868854 | 36.546 | 314.83 |
| $t=50^{\circ} \mathrm{C}$ |  |  |  |
| 0 | 0.912656 | 44.859 | 428.72 |
| 100 | 0.908625 | 43.685 | 428.71 |
| 200 | 0.904715 | 42.569 | 429.08 |
| 300 | 0.900921 | 41.506 | 429.75 |
| 400 | 0.897235 | 40.494 | 430.66 |
| 500 | 0.893652 | 39.529 | 431.74 |
| 600 | 0.890168 | 38.608 | 432.95 |
| 700 | 0.886777 | 37.730 | 434.22 |
| 800 | 0.883475 | 36.890 | 435.52 |
| 900 | 0.880257 | 36.089 | 436.80 |
| 1000 | 0.877120 | 35.322 | 438.02 |
| $t=75{ }^{\circ} \mathrm{C}$ |  |  |  |
| 0 | 0.924576 | 46.121 | 602.54 |
| 100 | 0.920384 | 44.789 | 593.65 |
| 200 | 0.916329 | 43.534 | 585.55 |
| 300 | 0.912403 | 42.351 | 578.16 |
| 400 | 0.908598 | 41.234 | 571.40 |
| 500 | 0.904907 | 40.179 | 565.22 |
| 600 | 0.901324 | 39.183 | 559.56 |
| 700 | 0.897842 | 38.240 | 554.37 |
| 800 | 0.894456 | 37.347 | 549.60 |
| 900 | 0.891159 | 36.503 | 545.21 |
| 1000 | 0.887948 | 35.702 | 541.16 |

$$
\begin{equation*}
V^{P}=V^{0}-V^{0} P /\left(B+A_{1} P+A_{2} P^{2}\right) . \tag{4}
\end{equation*}
$$

Differentiation of Eq. (4) with respect to pressure gives the compressibility (in $\mathrm{bar}^{-1}$ ):

$$
\begin{equation*}
\beta=\frac{-1}{V^{P}}\left(\frac{\partial V^{P}}{\partial P}\right)_{T}=\frac{V^{0}\left(B-A_{2} P^{2}\right)}{V^{P}\left(B+A_{1} P+A_{2} P^{2}\right)^{2}} . \tag{5}
\end{equation*}
$$

Differentiation of Eq. (4) with respect to temperature gives the expansibility (in $\mathrm{deg}^{-1}$ ):

$$
\begin{align*}
\alpha= & \frac{1}{V^{P}}\left(\frac{\partial V^{P}}{\partial T}\right)_{P}=\frac{1}{V^{P}}\left(\frac{\partial V^{0}}{\partial T}\right)-\frac{P\left(\partial V^{0} / \partial T\right)}{V^{P}\left(B+A_{1} P+A_{2} P^{2}\right)} \\
& +P V^{0} \frac{(\partial B / \partial T)+P\left(\partial A_{1} / \partial T\right)+P^{2}\left(\partial A_{2} / \partial T\right)}{V^{P}\left(B+A_{1} P+A_{2} P^{2}\right)^{2}} \tag{6}
\end{align*}
$$

For selected temperatures and pressures, we have tabulated in Table II the specific volumes, compressibilities, and expansibilities derived from our equation of state. These values are precise to within $\pm 15$ $\times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ in $V^{P}, \pm 2 \times 10^{-6} \mathrm{deg}^{-1}$ in $\alpha$, and $\pm 0.016$ $\times 10^{-6} \mathrm{bar}^{-1}$ in $\beta$. In Sec. III, these $P-V-T$ data are compared to other published results.

## III. COMPARISON OF RESULTS

Since few studies were made on the high pressure $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$, it is not possible to make a number of comparisons to our sound-derived data with direct measurements. Many workers (c.f. Refs. 5, 7, $8,10,13,14,16,18)$ measured or derived equations for the specific volume (or density) of $\mathrm{D}_{2} \mathrm{O}$ at 1 atm . We prefer the equation of $\mathrm{Kell}^{28}$ owing to the reported estimated error of $\pm 3 \times 10^{-6} \mathrm{~g} \mathrm{~cm}^{-3}$ and established accuracy of $\pm 10 \times 10^{-6} \mathrm{~g} \mathrm{~cm}^{-3}$. Also, many of the above mentioned studies do not cover as comprehensive a temperature range.

The most reliable 1 atm compressibilities for $\mathrm{D}_{2} \mathrm{O}$ are from the work of Millero and Lepple. ${ }^{6}$ Using a piezometric technique, they measured the compressibility of $\mathrm{D}_{2} \mathrm{O}$ from 5 to $65^{\circ} \mathrm{C}$ near 1 atm to within $\pm 0.1$ $\times 10^{-6} \mathrm{bar}^{-1}$. Comparisons of the 1 atm compressibilities determined from our equation of state and the work of Millero and Lepple ${ }^{6}$ are shown in Table III. The comparisons show that except at $65^{\circ} \mathrm{C}$ our results are in excellent agreement (average deviation $\pm 0.05_{5} \times 10^{-6}$ $\mathrm{bar}^{-1}$ ) with those of Millero and Lepple.

A number of workers ${ }^{17-20}$ examined the high pressure $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$. The most precise study at low temperatures is the work of Emmet and Millero. ${ }^{17}$ They measured the specific volumes of $(99.8 \%) \mathrm{D}_{2} \mathrm{O}$ (precise to within $\pm 10 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ ) from 2 to $40^{\circ} \mathrm{C}$ with a high pressure magnetic float densimeter. In the range of our equation of state $\left(5-100^{\circ} \mathrm{C}\right.$ and $\left.0-1000 \mathrm{bar}\right)$, the measurements of Juza et al., ${ }^{18}$ Kesselman, ${ }^{19}$ and Bridgman ${ }^{20}$ provide few data points for comparison. In Fig. 2 we compare the differences in the specific volumes of $\mathrm{D}_{2} \mathrm{O}$ obtained from our equation of state and

TABLE III. A comparison of the compressibility data ( $\mathrm{bar}^{-1}$ ) from the equation of state and Millero and Lepple ${ }^{6}$ at 1 atm.

|  |  | $\beta \times 10^{6}$ <br> (Millero | $\Delta \beta \times 10^{6}$ <br> [Eq. (5)-Millero <br> and Lepple) |
| :--- | :--- | :--- | :--- |
| $t\left({ }^{\circ} \mathrm{C}\right)$ | and Lepple] |  |  |
| 5 | $51.54_{7}$ | 51.49 | 0.06 |
| 10 | $49.80_{2}$ | 49.74 | 0.06 |
| 15 | $48.41_{7}$ | 48.38 | 0.04 |
| 20 | $47.32_{6}$ | 47.37 | -0.04 |
| 25 | 46.48 | 46.52 | -0.04 |
| 30 | $45.84_{1}$ | 45.88 | -0.04 |
| 35 | $45.37_{9}$ | 45.37 | 0.01 |
| 40 | $45.07_{3}$ | 45.10 | -0.03 |
| 45 | $44.90_{5}$ | 44.97 | -0.06 |
| 50 | $44.85_{9}$ | 44.91 | -0.05 |
| 55 | $44.92_{4}$ | 44.98 | -0.06 |
| 60 | 45.09 | 45.16 | -0.07 |
| 65 | $45.34_{9}$ | 45.51 | -0.16 |

V (Ours) -V (Emmet and Millero)


FIG. 2. A comparison of the differences in the specific volumes between the sound derived equation of state and the data of Emmet and Millero. ${ }^{17}$ Unit of contour: $10 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$.
the work of Emmet and Millero. The average deviation of the specific volume differences is $28 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$, and over $90 \%$ of the range ( $0-40^{\circ} \mathrm{C}, 0-1000 \mathrm{bar}$ ) the deviations are within $40 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$. The maximum deviation is $75 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ at $40^{\circ} \mathrm{C}$ and 1000 bar. The larger deviations at the high temperatures and pressures are probably a result of errors in the direct measurements due to the nonequilibrium of the magnetic float. ${ }^{31}$ The overall agreement is good and provides support for the validity of our equation of state.

Using a sylphon method, Bridgman ${ }^{20}$ measured the specific volume of $99.9 \% \mathrm{D}_{2} \mathrm{O}$ from -20 to $100^{\circ} \mathrm{C}$ and 0 to $\sim 12000$ bar. A comparison of our results (adjusted to $99.9 \% \mathrm{D}_{2} \mathrm{O}$ ) with those of Bridgman are shown in Table IV. The agreement is not very good; however, over most of the range the differences are within the experimental error of Bridgman's direct measurements $\left(\sim 500 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}\right)$. Kesselman ${ }^{19}$ derived an equation of state for $\mathrm{D}_{2} \mathrm{O}$ for the range $20-380^{\circ} \mathrm{C}$ and $0-500 \mathrm{bar}$. He states that the average deviations of his equation do not exceed the experimental error of Bridgman ${ }^{20}$ and Kirillin and Ulybin. ${ }^{32}$ As a consequence, the agreement between Kesselman's and our equation of state is very poor ( $\sim 1000 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ in specific volume). Poor agreement ( $1500 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ ) was also found with the data of Juza et al. ${ }^{18}$ They measured the specific volume of $\mathrm{D}_{2} \mathrm{O}$ at 80 and $100^{\circ} \mathrm{C}, 499$ and 999 bar.

## IV. COMPARISONS OF THE P-V-T PROPERTIES OF $\mathrm{D}_{2} \mathrm{O}$ AND $\mathrm{H}_{2} \mathrm{O}$

Since the viscosity, melting point, temperature of maximum density, and heat capacity are all higher in $\mathrm{D}_{2} \mathrm{O}$ than in $\mathrm{H}_{2} \mathrm{O}$, Nemethy and Scheraga ${ }^{33}$ (and others) proposed that there is more structural order in $\mathrm{D}_{2} \mathrm{O}$ than in $\mathrm{H}_{2} \mathrm{O}$, or that the degree of hydrogen bonding is higher in $\mathrm{D}_{2} \mathrm{O}$ than in $\mathrm{H}_{2} \mathrm{O}$. In recent years a number of workers ${ }^{21-23}$ have examined the structural properties of $\mathrm{D}_{2} \mathrm{O}$ relative to $\mathrm{H}_{2} \mathrm{O}$. Although the high pressure $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ are similar, the small differences that do occur can be very useful in

TABLE IV. A comparison of the specific volume data from the equation of state and Bridgman. ${ }^{20}$

|  | $\Delta V \times 10^{6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ |  |  |
| ---: | :---: | :---: | :---: |
| $t\left({ }^{\circ} \mathrm{C}\right)$ | 0 bar | 499 bar | 999 bar |
| 20 | -455 | -1045 | -1569 |
| 40 | 226 | -562 | -1027 |
| 50 | 262 | 270 | -1037 |
| 60 | -113 | -350 | -990 |
| 80 | 62 | -40 | -993 |
| 100 | $\cdots$ | 272 | -176 |

the examination of the structure of $\mathrm{H}_{2} \mathrm{O}$. It is not the purpose of this paper to discuss these differences in great detail; however, it is useful to examine some of them. Above the temperature of maximum density the $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ are similar. For example, the specific volumes increase as the temperature is increased, and decrease as the pressure is increased; the expansibilities increase as the temperature is increased and increase as the pressure is increased; the compressibilities decrease as the pressure is increased, and go through a minimum when plotted versus temperature. In this section we examine the effect of temperature and pressure on the $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$. The $P-V-T$ properties for $\mathrm{D}_{2} \mathrm{O}$ are from the equation of state (3), and the properties for $\mathrm{H}_{2} \mathrm{O}$ are from the equation derived by Fine and Millero. ${ }^{2}$

The effect of temperature on the specific volume, compressibility, expansibility, and specific heats at constant volume and pressure of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ are examined graphically. In Fig. 3 the specific volumes of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ are plotted versus temperature at two pressures ( $P=0$ and 1000 bar ). The curves are similar for $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ with $\mathrm{H}_{2} \mathrm{O}$ having the larger volumes. The compressibilities of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ (Fig. 4) show the minimum in the middle temperature range at both pressures ( 0 and 1000 bar).

The expansibilities of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ (Fig. 5) plotted versus temperature are also graphically similar. For both liquids the expansibility values at 0 and 1000 bar converge in the middle temperature range. The conver-


FIG. 3. The specific volumes of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ as a function of temperature at 0 and 1000 bar pressure.

